

# A New Model for Multiple-Winding Transformer

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*Abstract -- A new transformer model for multiple-winding transformer, which describes the leakage phenomenon, is presented. The unique feature of the model is that each parameter in the model corresponds to a physical magnetic flux. The model parameters can be determined by using the finite element tool. The model can also be converted into the other form, such as  $\pi$ -model.*

## I. Introduction

Multiple-winding transformers are often used in power electronics circuits. A typical application is multiple-output converters, where the multiple outputs are obtained by coupling several windings on the same transformer core. Since the cross regulation characteristics of this type converters are closely related to the transformer leakage inductances, modeling of leakage phenomenon is critical when analyzing multiple-output converters.

Transformer modeling has been an extensively researched topic. Much work has been done, which laid a good foundation for the application of the multiple-winding transformers. For a two-winding transformer, the  $\pi$ -model can be used to describe the transformer characteristics [1]. The  $\pi$ -model was also extended to three-winding transformers [2,3]. To model the transformer with more than four windings, the reluctance model [4] and cross-coupled model [5] were developed.

This paper presents a new model for multiple-winding transformer for the applications where precise modeling of leakage inductances is critical to the converter operation. The new model takes into account of the geometry of the magnetic structure while providing a unified expression. Each parameter in the

model has its physical meaning, and can be numerically determined for certain type of structures. The combinations of certain parameters can be analytically calculated and measured. Although the resulted model is difficult to use analytically, it can be used in conjunction with circuit simulators, such as PSpice or Saber, to provide more accurate simulation results.

## II. Derivation of the New Transformer Model

Figure 1 shows a three-winding transformer. Each winding carries a current with a voltage applied across its terminals. The interaction of the currents and voltages results in a magnetic field. The magnetic flux in the field can be decomposed into: (1) main flux,  $\Phi_m$ , which links all the windings; (2) self-leakage inductances,  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$ , and each of them links each winding itself; and mutual leakage inductances,  $\Phi_{12}$ ,  $\Phi_{13}$ , and  $\Phi_{23}$ , and each of them links two of the three windings as their subscripts denoted. The voltage across each winding can be expressed, in terms of the magnetic fluxes, as:

$$v_1 = N_1 \frac{d}{dt} (\Phi_m + \Phi_1 + \Phi_{12} + \Phi_{13}), \quad (1)$$

$$v_2 = N_2 \frac{d}{dt} (\Phi_m + \Phi_2 + \Phi_{12} + \Phi_{23}), \quad (2)$$

$$v_3 = N_3 \frac{d}{dt} (\Phi_m + \Phi_3 + \Phi_{13} + \Phi_{23}), \quad (3)$$

where the magnetic fluxes are the functions of the excitation currents as expressed as following:

$$\Phi_1 = P_1 N_1 i_1, \quad (4)$$

$$\Phi_2 = P_2 N_2 i_2, \quad (5)$$

$$\Phi_3 = P_3 N_3 i_3, \quad (6)$$

$$\Phi_{12} = P_{12} (N_1 i_1 + N_2 i_2), \quad (7)$$

$$\Phi_{23} = P_{23} (N_2 i_2 + N_3 i_3), \quad (8)$$

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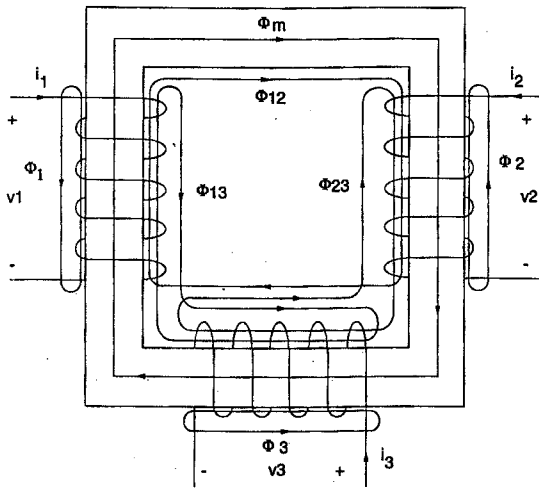


Fig. 1. A three-winding transformer. The magnetic flux can be decomposed into the main flux, which links all the windings and is mainly confined inside the core, and leakage fluxes, which link only one or two of three windings.

$$\Phi_{13} = P_{13} (N_3 i_1 + N_1 i_1), \quad (9)$$

where  $P_i$ 's and  $P_{ij}$ 's are the permeance of the magnetic paths of  $\Phi_i$  and  $\Phi_{ij}$ .

The main flux is confined inside the core and can be described:

$$\Phi_m = B A_c, \quad (10)$$

$$H l_m = N_1 i_1 + N_2 i_2, \quad (11)$$

$$B = \mu H, \quad (12)$$

then

$$\Phi_m = P_m (N_1 i_1 + N_2 i_2 + N_3 i_3), \quad (13)$$

where  $P_m$  is the permeance of the path of the main flux  $\Phi_m$ .

The Eqs. (1) - (3) can be written in the form of:

$$\begin{aligned} v_1 &= N_1 \frac{d}{dt} [P_m (N_1 i_1 + N_2 i_2 + N_3 i_3) + P_1 N_1 i_1 + \\ & P_{12} (N_1 i_1 + N_2 i_2) + P_{13} (N_1 i_1 + N_3 i_3)] \\ &= N_1^2 (P_m + P_1 + P_{12} + P_{13}) \frac{di_1}{dt} + N_1 N_2 (P_m \\ & + P_{12}) \frac{di_2}{dt} + N_1 N_3 (P_m + P_{13}) \frac{di_3}{dt} \\ v_2 &= N_2 \frac{d}{dt} [P_m (N_1 i_1 + N_2 i_2 + N_3 i_3) + P_2 N_2 i_2 \\ & + P_{12} (N_1 i_1 + N_2 i_2) + P_{23} (N_2 i_2 + N_3 i_3)] \end{aligned} \quad (14)$$

$$\begin{aligned} &= N_1 N_2 (P_m + P_{12}) \frac{di_1}{dt} + N_1^2 (P_m + P_2 \\ & + P_{12} + P_{23}) \frac{di_2}{dt} + N_2 N_3 (P_m + P_{23}) \frac{di_3}{dt}, \end{aligned} \quad (15)$$

$$\begin{aligned} v_3 &= N_3 \frac{d}{dt} [P_m (N_1 i_1 + N_2 i_2 + N_3 i_3) + P_3 N_3 i_3 \\ & + P_{13} (N_1 i_1 + N_3 i_3) + P_{23} (N_2 i_2 + N_3 i_3)] \\ &= N_1 N_3 (P_m + P_{13}) \frac{di_1}{dt} + N_2 N_3 (P_m + P_{23}) \frac{di_2}{dt} \\ & + N_3^2 (P_m + P_3 + P_{13} + P_{23}) \frac{di_3}{dt} \end{aligned} \quad (16)$$

Referring all the voltages and currents to winding 1, i.e.,

$$v_2 = \frac{N_2}{N_1} v_{2-1}, \quad (17)$$

$$i_2 = \frac{N_1}{N_2} i_{2-1}, \quad (18)$$

$$v_3 = \frac{N_3}{N_1} v_{3-1}, \quad (19)$$

$$i_3 = \frac{N_1}{N_3} i_{3-1}, \quad (20)$$

and substituting the above relations into Eqs. (14) - (16) then the following expressions can be obtained:

$$\begin{aligned} v_1 &= N_1^2 (P_m + P_1 + P_{12} + P_{13}) \frac{di_1}{dt} + N_1^2 (P_m \\ & + P_{12}) \frac{di_{2-1}}{dt} + N_1^2 (P_m + P_{13}) \frac{di_{3-1}}{dt} \\ &= M_o^1 \frac{di_1}{dt} + L_{11}^1 \frac{di_1}{dt} + L_{112}^1 \frac{di_1}{dt} + L_{113}^1 \frac{di_1}{dt} + M_o^1 \frac{di_{2-1}}{dt} \\ & + L_{112}^1 \frac{di_{2-1}}{dt} + M_o^1 \frac{di_{3-1}}{dt} + L_{113}^1 \frac{di_{3-1}}{dt}, \end{aligned} \quad (21)$$

$$\begin{aligned} v_{2-1} &= N_1^2 (P_m + P_{12}) \frac{di_1}{dt} + N_1^2 (P_m + P_2 \\ & + P_{12} + P_{23}) \frac{di_{2-1}}{dt} + N_1^2 (P_m + P_{23}) \frac{di_{3-1}}{dt} \\ &= M_o^1 \frac{di_1}{dt} + L_{112}^1 \frac{di_1}{dt} + M_o^1 \frac{di_{2-1}}{dt} + L_{112}^1 \frac{di_{2-1}}{dt} \\ & + L_{112}^1 \frac{di_{2-1}}{dt} + L_{123}^1 \frac{di_{2-1}}{dt} + M_o^1 \frac{di_{3-1}}{dt} + L_{123}^1 \frac{di_{3-1}}{dt}, \end{aligned} \quad (22)$$

$$\begin{aligned} v_{3-1} &= N_1^2 (P_m + P_{13}) \frac{di_1}{dt} + N_1^2 (P_m + P_2 \\ & + P_{13} + P_{23}) \frac{di_{3-1}}{dt} \end{aligned}$$



Table 2. Calculated Magnetic and Leakage Inductances.

$M_o^1(\mu H)$	$L_{11}^1(nH)$	$L_{12}^1(nH)$
20.179	8.296	0
$L_{13}^1(nH)$	$L_{112}^1(nH)$	$L_{123}^1(nH)$
23.68	6.414	29.875

Generally speaking, the new transformer model has  $n(n+1)/2$  leakage inductances, where  $n$  is the number of the windings. For a specific magnetic structure, however, the number of the parameters can be less. In the three-winding transformer with side-by-side arrangement, there are only five parameters instead of six. In this structure, winding 2 is sandwiched by windings 1 and therefore any flux which links winding 2 must link either winding 1 or winding 3. This means there is no  $L_{12}$ . In addition, any flux which links windings 1 and 3, must also link winding 2. According to the definition of each flux component, this is the main flux. So there is no leakage flux which links windings 1 and 3,  $M_{113}$ . The model of three-winding transformer with side-by-side structure is shown in Fig. 4.

Since leakage inductances can only be accurately measured by performing short circuit test, it is impossible to measure the model parameters individually. Nonetheless, it is possible to measure certain combination of the leakage inductances. Table 3 shows the measurement and calculation results.

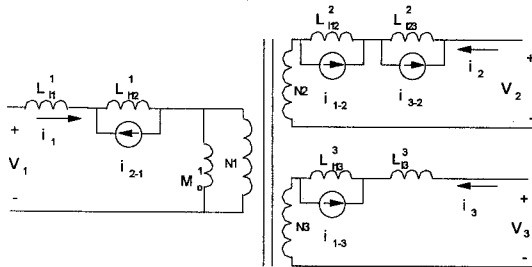


Fig. 4. Three-winding transformer with side-by-side winding structure. In the general form of the three-winding transformer, there are six leakage inductances. For the transformer with side-by-side winding arrangement, only four leakage inductances exist.

It can be seen that the calculated results are fairly close to the experimental results. The discrepancies are mainly due to the difference between the physical and modeled geometries, such as the terminations. Besides, the modeling tool is a 2-D solver, but the real problem is in nature a 3-D problem, which also accounted for the calculation errors.

Table 3. Measurement and Calculation of the Combination of the Leakage Inductances.

	$L_{1s2}(nH)$	$L_{1s3}(nH)$	$L_{2s3}(\mu H)$
Calculation	38.171	68.265	3.161
Measurement	42.67	81.76	3.22

Note:  $L_{isj}$  stands for the inductance, either measured or calculated, at  $i$  winding with  $j$  winding shorted.

Figure 5 shows the geometrical structure of a four-winding transformer. The model for a four-winding transformer with side-by-side structure is shown in Fig. 6. It can be seen that the real model takes much simpler form than the general model.

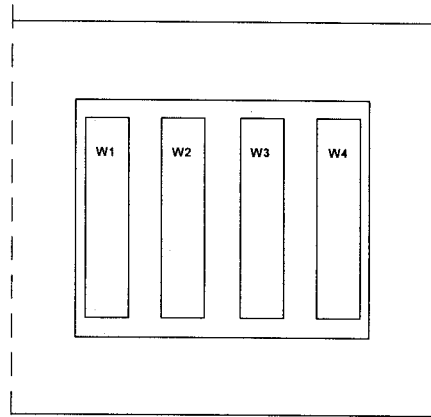


Fig. 5. A four-winding transformer with side-by-side winding structure.

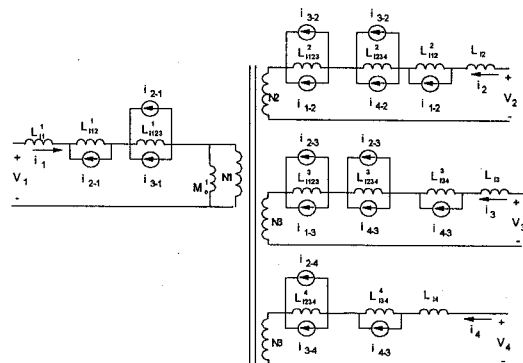


Fig. 6. The final model for a four-winding transformer with side-by-side winding structure. Due to the specific geometry, many components in the general model no longer exist. As a result, the model is simplified and more manageable.

#### IV. Conversion of the New Transformer Model into Other Forms

The new transformer can be easily written in other forms. It is demonstrated here how to convert the 3-winding transformer model into a  $\pi$ -model which was originally presented in [2], and the derivation was provided in [3]. It can be seen that the  $\pi$ -model of the three-winding transformer is only a special case of the new model. According to the new model, the port voltages for the transformer can be expressed:

$$v_1 = (M_o^l + L_{11}^l + L_{112}^l + L_{113}^l) \frac{di_1}{dt} + (M_o^l + L_{112}^l) \frac{N_2}{N_1} \frac{di_2}{dt} + (M_o^l + L_{113}^l) \frac{N_3}{N_1} \frac{di_3}{dt}, \quad (47)$$

$$v_2 = (M_o^l + L_{112}^l) \frac{N_2}{N_1} \frac{di_1}{dt} + (M_o^l + L_{12}^l + L_{112}^l + L_{123}^l) \frac{N_2}{N_1^2} \frac{di_2}{dt} + (M_o^l + L_{123}^l) \frac{N_2 N_3}{N_1^2} \frac{di_3}{dt}, \quad (48)$$

$$v_3 = (M_o^l + L_{113}^l) \frac{N_3}{N_1} \frac{di_1}{dt} + (M_o^l + L_{123}^l) \frac{N_2 N_3}{N_1^2} \frac{di_2}{dt} + (M_o^l + L_{13}^l + L_{113}^l + L_{123}^l) \frac{N_3}{N_1^2} \frac{di_3}{dt}. \quad (49)$$

As mentioned before, a three-winding transformer can be put into standard coupled-inductor form:

$$v_1 = L_{11} \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt}, \quad (50)$$

$$v_2 = M_{12} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt} + M_{23} \frac{di_3}{dt}, \quad (51)$$

$$v_3 = M_{13} \frac{di_1}{dt} + M_{23} \frac{di_2}{dt} + L_{33} \frac{di_3}{dt}, \quad (52)$$

where the self and mutual inductances can be expressed by the parameters in the model:

$$L_{11} = M_o^l + L_{11}^l + L_{112}^l + L_{113}^l, \quad (53)$$

$$L_{22} = \frac{N_2^2}{N_1^2} (M_o^l + L_{12}^l + L_{112}^l + L_{123}^l), \quad (54)$$

$$L_{33} = \frac{N_3^2}{N_1^2} (M_o^l + L_{13}^l + L_{113}^l + L_{123}^l), \quad (55)$$

$$M_{12} = \frac{N_2}{N_1} (M_o^l + L_{112}^l), \quad (56)$$

$$M_{13} = \frac{N_3}{N_1} (M_o^l + L_{113}^l), \quad (57)$$

$$M_{23} = \frac{N_2 N_3}{N_1^2} (M_o^l + L_{123}^l), \quad (58)$$

Using these values, the parameters in the corresponding  $\pi$ -model can be calculated:

$$\begin{aligned} L_1 &= L_{11} - \frac{M_{12} M_{13}}{M_{23}} = (M_o^l + L_{11}^l + L_{112}^l + L_{113}^l) \\ &\quad - \frac{\frac{N_2}{N_1} (M_o^l + L_{112}^l) \frac{N_3}{N_1} (M_o^l + L_{113}^l)}{\frac{N_2 N_3}{N_1^2} (M_o^l + L_{123}^l)} \\ &= \frac{M_o^l L_{11}^l + M_o^l L_{123}^l + L_{11}^l L_{123}^l + L_{112}^l L_{123}^l}{M_o^l + L_{123}^l} \\ &\quad - \frac{+L_{113}^l L_{123}^l - L_{112}^l L_{113}^l}{M_o^l + L_{123}^l} L_{11}^l + L_{123}^l \end{aligned} \quad (59)$$

$$\begin{aligned} L_2 &= L_{22} - \frac{M_{12} M_{23}}{M_{13}} = \frac{N_2^2}{N_1^2} (M_o^l + L_{11}^l + L_{112}^l + L_{113}^l) \\ &\quad - \frac{\frac{N_2}{N_1} (M_o^l + L_{112}^l) \frac{N_2 N_3}{N_1^2} (M_o^l + L_{123}^l)}{\frac{N_3}{N_1} (M_o^l + L_{113}^l)} \\ &= \frac{N_2^2}{N_1^2} \frac{M_o^l L_{12}^l + M_o^l L_{113}^l + L_{12}^l L_{113}^l + L_{112}^l L_{113}^l}{M_o^l + L_{113}^l} \\ &\quad - \frac{+L_{123}^l L_{113}^l - L_{112}^l L_{123}^l}{M_o^l + L_{113}^l} \frac{N_2^2}{N_1^2} (L_{12}^l + L_{113}^l) \end{aligned} \quad (60)$$

$$\begin{aligned} L_3 &= L_{33} - \frac{M_{23} M_{13}}{M_{12}} = \frac{N_3^2}{N_1^2} (M_o^l + L_{11}^l + L_{112}^l + L_{113}^l) \\ &\quad - \frac{\frac{N_2 N_3}{N_1^2} (M_o^l + L_{123}^l) \frac{N_3}{N_1} (M_o^l + L_{113}^l)}{\frac{N_2}{N_1} (M_o^l + L_{112}^l)} \\ &= \frac{N_3^2}{N_1^2} \frac{M_o^l L_{13}^l + M_o^l L_{112}^l + L_{123}^l L_{112}^l + L_{112}^l L_{113}^l}{M_o^l + L_{112}^l} \\ &\quad - \frac{+L_{123}^l L_{112}^l - L_{123}^l L_{113}^l}{M_o^l + L_{112}^l} \frac{N_3^2}{N_1^2} (L_{13}^l + L_{112}^l) \end{aligned} \quad (61)$$

$$M = \frac{M_{12}M_{13}}{M_{23}} = \frac{\frac{N_2}{N_1}(M_o^I + L_{112}^I) \frac{N_3}{N_1}(M_o^I + L_{113}^I)}{\frac{N_2N_3}{N_1^2}(M_o^I + L_{123}^I)} \approx M_o \quad (62)$$

$$A = \frac{M_{23}}{M_{13}} = \frac{\frac{N_2N_3}{N_1^2}(M_o^I + L_{123}^I)}{\frac{N_3}{N_1}(M_o^I + L_{113}^I)} \approx \frac{N_2}{N_1} \quad (63)$$

$$B = \frac{M_{23}}{M_{12}} = \frac{N_3}{N_1} \frac{\frac{N_2N_3}{N_1^2}(M_o^I + L_{123}^I)}{\frac{N_2}{N_1}(M_o^I + L_{112}^I)} \approx \frac{N_3}{N_1} \quad (64)$$

The equivalent circuit is shown in Fig. 7. Again if the windings are arranged in some regular way, each parameter can be estimated by either analytical or numerical calculation.

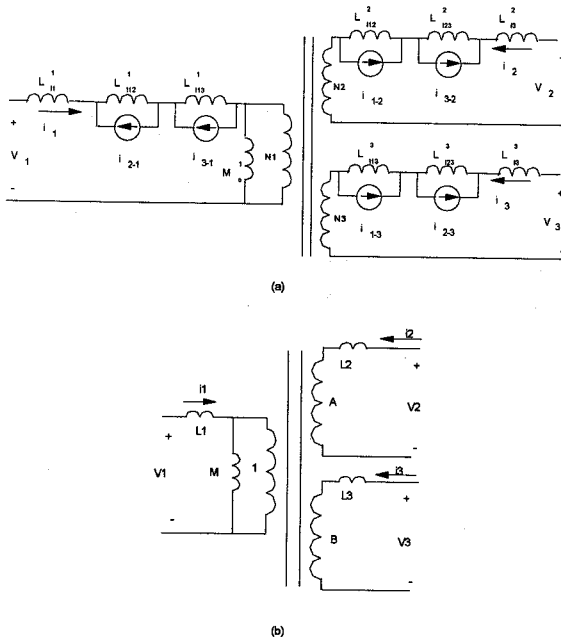


Fig. 7. Conversion of the new model to the  $\pi$ -model for a three-winding transformer. The  $\pi$ -model is actually a special case of the new model when the number of the windings is three.

If a three-winding transformer structure has a

specific magnetic structure, e.g., the windings are arranged side by side, the reluctance model presented in [4] can be formulated. The following shows this reluctance model can also be obtained from the new transformer model. Substituting these parameters into Eqs. (59) - (61), the three leakage inductances are

$$L_1 \approx L_{11}^I + L_{123}^I \quad (65)$$

$$L_2 \approx \frac{N_2^2}{N_1^2}(L_{12}^I + L_{113}^I) = 0 \quad (66)$$

$$L_3 \approx \frac{N_3^2}{N_1^2}(L_{13}^I + L_{112}^I) \quad (67)$$

Constructing a circuit model corresponding to (65) - (67), it is same as the one given in [4].

## V. Summary

A new model for multiple-winding transformers is presented. The model takes very general form regardless of geometry and magnetic property. The derivation starts from the Faraday's law, which relates the magnetic fluxes to the voltage across each winding. Then each flux is expressed in terms of the current through windings. Although the model takes the most general form, the calculation of the model parameters has to be performed for specific magnetic structure with the help of numerical simulation tool -- finite element method. The unique feature of the model is that each parameter in the model corresponds to a physical flux in the transformer, which distinguishes it from the existing models. The certain combinations of the leakage inductances can be measured. The calculated and experimental results are very close.

The problem related to the new model is that it is relatively complicated and difficult to use when performing analytical calculation. However, it can be useful for circuit simulation.

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