SMALL-SIGNAL MEASUREMENT TECHNIQUES IN SWITCHING POWER SUPPLIES

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Abstract - **As the power supply dynamic specifications become tighter, the control bandwidth has to be increased which interferes with the stability. Therefore, the feedback design becomes critical and should be optimized. In many practical cases, the accuracy of the power stage small-signal model is not enough for the adequate design of the loop gain. The optimal feedback design requires measurement of the plant transfer function and of the loop gain. The limitations of the conventional closed-loop measurement setup are shown and a measurement procedure, which is free of those limitations, is proposed. The plant identification in several common types of power supplies is non-trivial and is discussed in detail. An example of the small-signal measurement in the multiple-loop computer power supply is presented. Finally, the loop gain measurement of unstable power supplies is considered.**

I. INTRODUCTION

Today's data-processing ICs operate from low-voltage power sources with tight output regulation tolerances. At the same time, those ICs represent highly dynamic loads for their power supplies. Therefore, the transient response requirements for supplies delivering power to the data-processing equipment are becoming tighter. To meet transient specifications, the control bandwidth of the power supplies often has to be increased which interferes with the system stability. Under these circumstances, the feedback design becomes critical.

The power supply control loop can be represented by a plant and an error amplifier (EA), as shown in Fig. 1(a). EA transfer function G_{EA} is designed based on plant transfer function G_{PL} to meet stability and transient response specifications. Commonly, the plant transfer function is calculated based on a continuous-time power stage small-signal model, which is available for the majority of power supply topologies. Generally, the small-signal models tend to be more accurate in a low-frequency range and less accurate (particularly, phase characteristic) in a high-frequency range due to a number of high-frequency phenomena which are difficult to account for [1]. With the control bandwidth increase, the limited accuracy of the small-signal models at high frequencies is becoming more and more critical. In many cases, the preliminary feedback design which is based only on the calculated plant transfer function is not sufficient to meet the specifications and requires additional optimization. The optimization procedure includes:

(1) measurement of plant transfer function G_{PL},
(2) design of transfer function G_{EA} based on the measured plant transfer function,
(3) final measurement of loop gain $T = G_{EA} \cdot G_{PL}$ for design verification.

The purpose of this paper is to present small-signal measurements techniques for power supplies which are generally considered difficult to measure. Sections II and III explain limitations of the conventional measurement setup and propose a more general procedure for measuring transfer functions and a loop gain. The success of the feedback design depends on the simple fact that the plant and EA must be properly identified in a real power supply. Although this identification appears trivial in some power supplies, in others it presents a serious challenge and is considered in Section IV. Section V provides an application example of the proposed small-signal measurement technique to the multiple-loop power supply for desktop computer. Section VI is dedicated to the measurement of transfer functions in unstable power supplies.

II. CONVENTIONAL SMALL-SIGNAL MEASUREMENT SETUP AND ITS LIMITATIONS

The loop gain and transfer functions are usually measured in the conventional closed-loop setup [2, 3], shown on the block diagram in Fig. 1(a). Excitation source $\boldsymbol{V_{AC}}$ is injected at the plant output, although it can be injected at the EA output. The loop gain $\boldsymbol{T}$ is measured as $\boldsymbol{T} = \hat{V}_X / \hat{V}_O$. The block diagram in Fig. 1(a) corresponds to the equivalent electrical circuit in Fig. 1(b), where Z_O represents the plant output impedance and Z_{IN} represents the EA input impedance. The remaining part of the feedback loop is represented by the transfer function $\boldsymbol{-T_X(s)}$ of the dependent voltage source $-T_X \cdot \hat{V}_X$. The actual gain of the feedback loop is

$$\mathbf{T = T_X \cdot Z_{IN} / (Z_O + Z_{IN})}. \qquad (1)$$

However, the loop gain, measured by a network analyzer as the ratio of the voltages V_{TEST} and V_{REF}, is derived as

$$T_M = \hat{V}_X / \hat{V}_X = T_X + Z_O / Z_{IN} . \quad (2)$$

For the valid measurement ($T_M = T$), two conditions have to be satisfied:

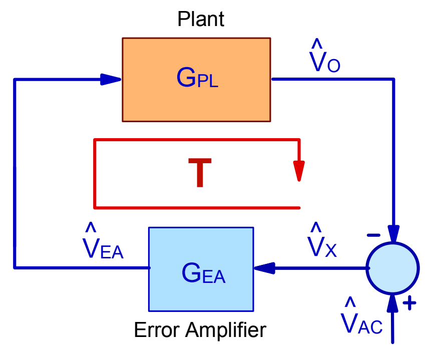
$$Z_O/Z_{IN} << 1 \quad (3)$$

and $$Z_O/Z_{IN} << T_X. \quad (4)$$

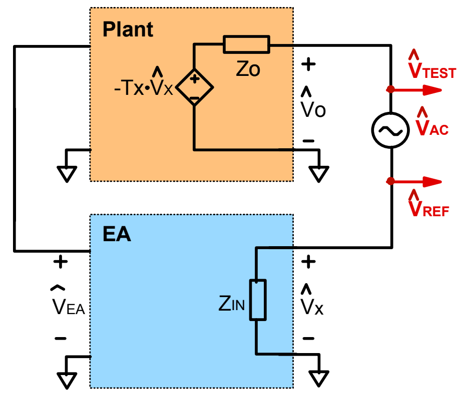
Otherwise, the measurement results obtained with the conventional setup are distorted. Although conditions (3) and (4) are satisfied in many practical cases, there are classes of power supplies where those conditions are violated. The multiple-loop supply for a desktop computer, presented in Section V, is an example of such power supply.

III. Proposed Small-Signal Measurement Setup

It is important to trace the origin of the distortion when conditions (3) and (4) are violated. It is evident from Fig. 1(b) that the EA transfer function $G_{EA} = \hat{V}_{EA} / \hat{V}_X$ is not affected by the insertion of the source $\hat{V}_{AC}$. However, the plant output signal $\hat{V}_O$ is determined by superposition of $-T_X \cdot \hat{V}_X$ and $\hat{V}_{AC}$ sources that causes the distortion of plant transfer function $G_{PL} = \hat{V}_O / \hat{V}_{EA}$. Understanding of the distortion origin leads to the more general measurement setup which does not depend on observation of (3) and (4). As previously mentioned, the setup in Fig. 1(b) produces the correct measurement of EA transfer function G_{EA} independently of observation of (3) and (4). Similarly, the correct measurement of plant transfer function G_{PL} can be obtained in the additional setup, shown in Fig. 1(c), where the excitation source $\hat{V}_{AC}$ is inserted at the EA output. In Fig. 1(c), the plant transfer function is measured as $\hat{V}_O / \hat{V}_Z$. Given the correct measurements of the plant and EA transfer functions, the loop gain is then computed as $\boldsymbol{T} = \boldsymbol{G_{EA}} \cdot \boldsymbol{G_{PL}}$. If the logarithmic scale was used for transfer function measurements, computation of loop gain $\boldsymbol{T}$ requires summing of the magnitudes and phases of transfer functions. Therefore, the proposed computation has low sensitivity to the measurement errors of the plant and EA transfer functions. The proposed loop gain measurement procedure is general since it does not depend on the relationship between Z_{IN} and Z_O. This generality is achieved at the expense of adding one more setup which is necessary for the measurement of the second loop-gain component. The proposed measurement procedure provides also a higher degree of freedom in terms of the signal injection. Namely, the signal V_{AC} can be injected in series with the plant and EA, as shown in Fig. 1(c). However, the measured plant transfer function does not change if the excitation source V_{AC} with impedance Z_{AC} and dc blocking capacitor C_B is injected in parallel with the plant and EA, as shown in Fig. 2.



(a)



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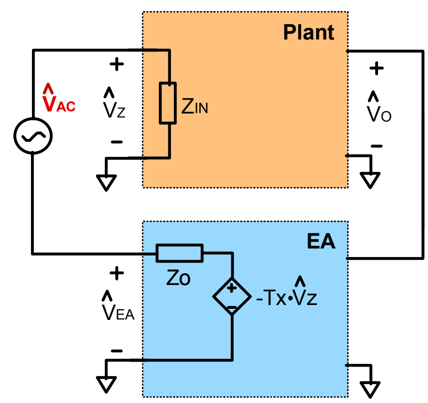


(c)

Fig. 1. Small-signal measurement setup: (a) block diagram; (b) conventional setup; (c) setup for plant transfer function measurement.

IV. Plant Transfer Function Identification And Measurement

Usually, the transfer function measurements are straightforward as long as the feedback signal is confined to a single path. However, when the feedback signal propagates through several paths, i.e., several feedback loops, the identification of the transfer functions which should be measured and applied to the EA design becomes difficult. From the general control theory, each loop gain of

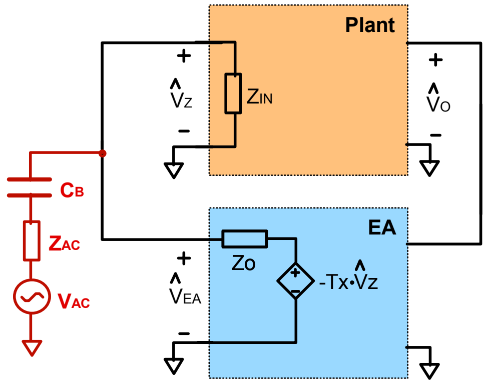


Fig. 2. Alternative connection of excitation source V_{AC} in plant transfer function measurement setup.

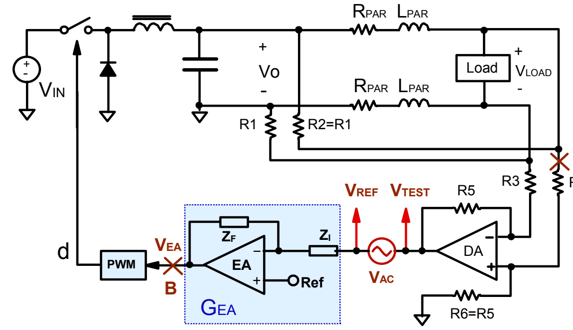


Fig. 3. Transfer function measurement in power supplies with remote voltage feedback.

the entire control system has the same characteristic polynomial and, therefore, can be used to judge system stability. However, different loop gains have an unequal value for the EA design as well as for prediction of the power supply rejection of the line and load disturbances. Generally, there are two major advantages of finding a location where the feedback signal is confined to a single path and breaking the loop at this location for the loop gain measurement.

(1) If the feedback signal is confined to a single path, only one loop gain has to be examined for stability. If the feedback signal is not confined to a single path, the other signal paths form internal loops. The stability of these loops has to be examined first, prior to loop gain measurement of the entire power supply. Then, the loop gain analysis becomes a two-stage procedure.

(2) If the feedback signal is confined to a single path, the relationship between measured loop gain and power supply rejection of the line and load disturbances is straightforward, particularly in power supplies with voltage-mode control [4].

Therefore, the focus of this section is to find the locations where the feedback signal is confined to a single path and propose measurement setups corresponding to these locations. Several typical power supplies are considered which require careful examination prior to identification of the critical loop gains that provide the basis for the EA design and determine the power supply closed-loop performance.

A. Power Supplies With Remote Feedback

A simplified circuit diagram of a typical power supply with remote voltage feedback is shown in Fig. 3. The difference amplifier DA compensates for voltage drops across parasitic ESR R_{PAR} and ESL L_{PAR} of the connecting cables. Resistors R_1 and R_2 provide the local feedback when the load is disconnected and are typically in the range of 10-100 Ω. Amplifier DA also introduces a gain $K_D = R_5 / R_3$ in the feedback path. In power supplies with local feedback, the excitation signal is commonly injected between the power supply output V_O and the impedance Z_I of the error amplifier. This location meets conditions (3) and (4), discussed in Section II. Therefore, it is tempting to inject the excitation signal at location A, as shown in Fig. 3. However, signal insertion at location A does not break the feedback path completely since the feedback signal still propagates through resistor R1. The excitation signal can also be injected at location B, between the output of error amplifier EA and PWM. However, the connection between the EA output and PWM is often inaccessible since it is inside the PWM IC, and signal insertion at this location cannot be implemented practically. Therefore, the only available location for the excitation source is at the output of DA. The plant and EA transfer functions are then identified as $G_{PL} = \hat{V}_{TEST} / \hat{V}_{EA}$ and $G_{EA} = -\hat{V}_{EA} / \hat{V}_{REF}$, respectively. Since the power supply has local and remote grounds, it is important to select the correct ground as a reference for the measured signals V_{TEST} and V_{REF}. The correct ground is the local one, since the feedback control signals are referenced with respect to it. From authors' experience, connection to the wrong ground can easily cause 10-20º phase measurement error.

B. Power Supplies with Magamp Control

A simplified circuit diagram of a power supply with magamp control is shown in Fig. 4. The reset circuit of saturable reactor SR includes transistor Q1, resistors R_S, R_B, R_E, and diodes D1, D2. The reset circuit is supplied from output voltage V_O. Figure 4 indicates that at location A the feedback signal has a single path and, therefore, it is meaningful to measure the loop gain associated with breaking the loop at this location. However, input impedance Z_{IN} of the reset circuit, shown in Fig. 4, is very low at low frequencies due to the high EA gain. Therefore, placement of the excitation source at location A violates input/output impedance relationship (3). Actually, for this type of magamp control, it is impossible to find a location in the control loop, which guarantees satisfaction of the impedance relationship (3) and where all feedback paths

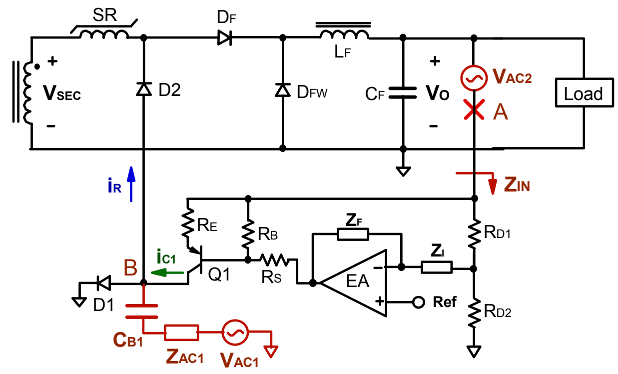


Fig. 4. Transfer function measurement in power supplies with magamp control.

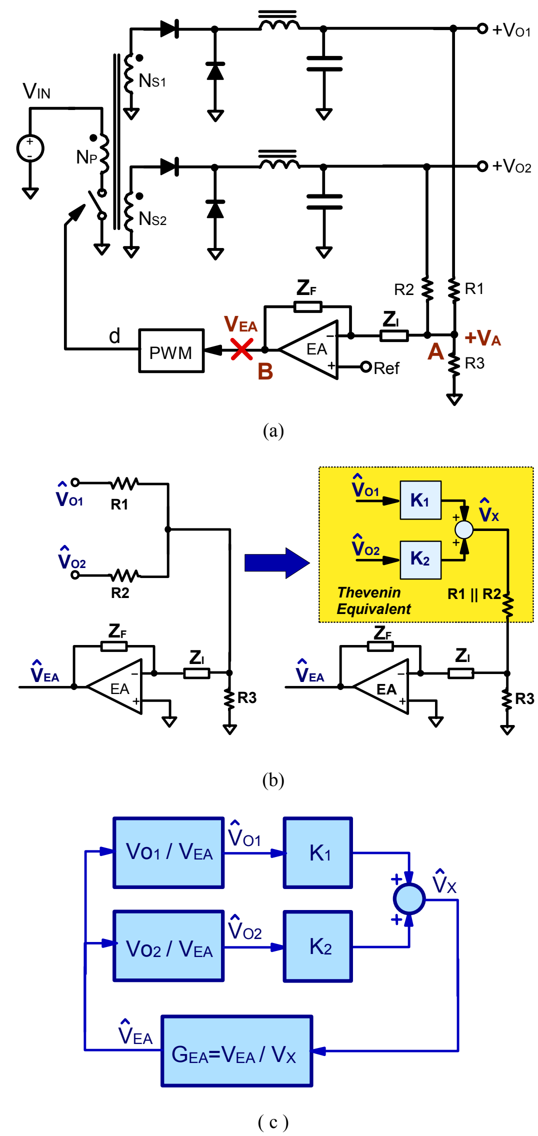


Fig. 5. Transfer function measurement in power supplies with weighted-mode control: (a) simplified circuit diagram; (b) derivation of transfer functions; (c) block diagram.

are broken.

However, the loop gain can be measured using the proposed setup, outlined in Section III. The plant and EA transfer functions are defined as $G_{PL} = \hat{V}_O / \hat{i}_{C1}$ and $G_{EA} = \hat{i}_{C1} / \hat{V}_O$, where i_{C1} is the collector current of Q1. Although the magamp is actually controlled by reset current i_R, this current is discontinuous and cannot be used in small-signal measurements. A practical measurement signal is Q1 collector current i_{C1} which is continuous and can be measured with a current transformer. To measure plant transfer function G_{PL}, excitation source V_{AC1} with impedance Z_{AC1} and blocking capacitor C_{B1} are placed at location B, as shown in Fig. 4. To measure EA transfer function G_{EA}, excitation source V_{AC2} is placed at location A shown in Fig. 4. Finally, the two components of the loop gain are summed (in the logarithmic scale) to produce the loop gain of magamp control.

C. Multioutput Power Supplies with Weighted Feedback

Weighted feedback is used to meet output regulation specifications in power supplies where the required regulation is not very tight, for instance, in desktop computer power supplies. The weighted feedback is a cost-effective solution since it uses a single power train to regulate several outputs. A simplified circuit diagram of a power supply with weighted-feedback control [4, 5] is shown in Fig. 5(a). The feedback loop actually regulates the weighted sum $K_1 \cdot V_{O1} + K_2 \cdot V_{O2}$, where $K_1 = R_2 \cdot R_3 / (R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3)$ and $K_2 = R_1 \cdot R_3 / (R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3)$. By assigning the larger weight K_1 for output V_{O1}, its regulation is improved at the expense of the output V_{O2} regulation.

It seems natural to break the loop at point A in Fig. 5(a), where the feedback signals from outputs V_{O1} and V_{O2} are summed. However, the output impedance of the network connected to point A is not low, and relationship (3) between the input and output impedances is often violated. Actually, impedance Z_I loads the network of resistors R_1, R_2, and R_3. Therefore, plant transfer function G_{PL} cannot be defined as $\hat{V}_A / \hat{V}_{EA}$, where $\hat{V}_A$ is the voltage of node A in Fig. 5(a).

To derive the plant transfer function, the network of output voltage sources V_{O1}, V_{O2} and resistors R_1, R_2, and R_3 is replaced by its Thevenin equivalent, as shown in Fig. 5(b). Then, the feedback loop can be represented by the block diagram in Fig. 5(c). Fig. 5(c) clearly indicates that the plant transfer function can be defined as $G_{PL} = K_1 \cdot \hat{V}_{O1} / \hat{V}_{EA} + K_2 \cdot \hat{V}_{O2} / \hat{V}_{EA}$. Hence, plant transfer function G_{PL} requires measurement of two separate transfer functions $\hat{V}_{O1} / \hat{V}_{EA}$ and $\hat{V}_{O2} / \hat{V}_{EA}$ and calculation of their weighted sum. Inspection of Fig. 5(b) also indicates that the compensator transfer function should be defined as $G_{PL} = -\hat{V}_{EA}/\hat{V}_X = -Z_F/[Z_I + (R_1 \| R_2 \| R_3)]$. To measure the loop gain, the excitation source should be injected at location B at the EA output, as shown in Fig. 5(a).

V. Loop Gain Measurement Example Using Proposed Measurement Setup

A feedback circuit diagram of the forward power supply with the 5-V and 12-V outputs for a desktop computer application is shown in Fig. 6. The power supply employs weighted feedback, as described in Section IV. The TL431 shunt regulator is supplied from the 12-V output. The networks Z_1, Z_2, Z_3, and Z_4 provide the compensation of the feedback loop. The power supply control loop does not have a location for V_{AC} injection where the feedback signal is confined to a single path and where at the same time impedance relationship (3) is satisfied. Hence, to measure the loop gain, the proposed measurement setup presented in Section III is applied. Loop gain T is measured as a product of two transfer functions $T = -G_1 \cdot G_2$, which are defined as $G_1 = \hat{i}_F/\hat{V}_X$ and $G_2 = \hat{V}_X/\hat{i}_F$ and where i_F is the LED forward current in Fig. 6. To measure transfer function G_1, excitation source V_{AC1} with resistor $R_{AC1} = 13$ kΩ and blocking capacitor $C_{B1} = 10$ µF is placed on the primary side, as shown in Fig. 6. To measure transfer function G_2, excitation source V_{AC2} with impedance $R_{AC2} = 13$ kΩ and blocking capacitor $C_{B2} = 10$ µF is placed on the secondary side. Optocoupler current i_F was measured with a current transformer. Measurement of transfer functions G_1 and G_2 produces loop gain T, shown in Fig. 7. To verify the proposed measurement method, loop gain T was also measured with a different approach [7], which requires insertion of a buffer and of impedance matching networks in order to achieve the impedance relationship (3). The measurement setup with a buffer is shown in Fig. 8. The setup assumes that the power supply employs a popular low-cost PWM controller which has a resistive 3:1 divider in front of the PWM comparator. The typical value of resistor R is 11.5 kΩ. To keep the original operating point of the feedback circuit, resistor of 3·R value and current source $I_X = I_{DC}$ are added on the buffer input side, as shown in Fig. 8. The excitation source V_{AC} is added at the buffer output and the loop gain is measured as the ratio $\hat{V}_{TEST}/\hat{V}_{REF}$.

Both measured loop gains are compared in Fig. 7. Observation of Fig. 7 shows good matching of the loop gains measured with two different methods. Although there are minor discrepancies between measured plots in the low-frequency and the high-frequency ranges, the plots

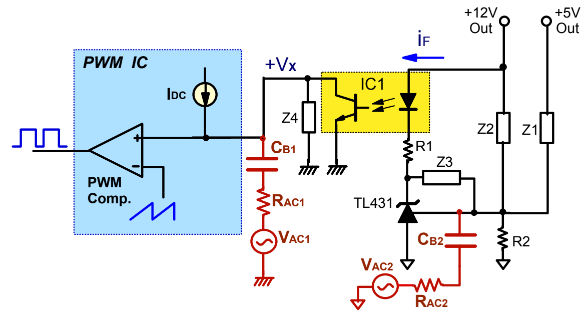


Fig. 6. Simplified feedback circuit diagram of desktop computer power supply

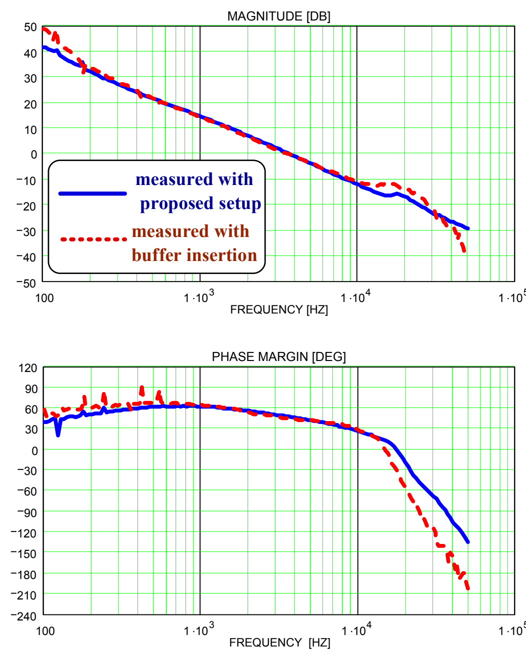


Fig. 7. Loop gain of the multiple-loop desktop computer power supply.

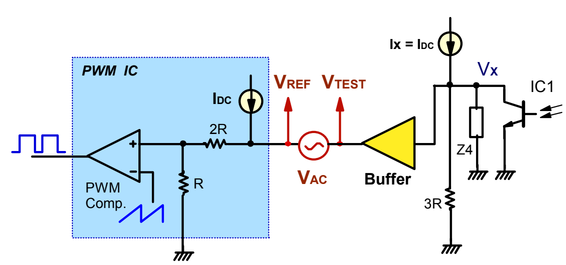


Fig. 7. Measurement of the computer power supply loop gain with the buffer.

show the same bandwidth and the same stability phase margin.

VI. Loop Gain Measurement of Unstable Power Supplies

When the power supply oscillates, it becomes even more important to measure its loop gain in order to find the oscillation origin. However, the small-signal measurement makes sense only in the vicinity of a stable operating point. Any measurement when the power supply is oscillating is meaningless. One way to stabilize the control loop is to insert an excitation source with high impedance Z_{AC} in series between the plant and EA [3]. As shown in Fig. 9, impedance Z_{AC} is a parallel connection of resistor R_{AC} and inductor L_X and is connected between output V_O and resistor R_{D1} of the voltage divider. The purpose of resistor R_{AC} is to decrease the effective gain of voltage divider K_D in the frequency range of interest, whereas inductor L_X maintains the dc operating point unchanged. To test the practicality of this method, the following example is considered. It is assumed that $R_{D1} = R_{D2} = 1$ kΩ. To reduce the loop gain by 6 dB, resistor R_{AC} of 2-kΩ value has to be added. It is further assumed that the measurement frequency range starts from $f_{MIN} = 10$ Hz. Therefore, at 10 Hz reactive impedance $X_L = 2\cdot\pi\cdot f_{MIN}\cdot L_X$ should be much higher than R_{AC} resistance. Assume impedance $X_L = 2\cdot\pi\cdot f_{MIN}\cdot L_X = 10\cdot R_X = \ = 20$ kΩ. The resulting L_X value is 318 H. This value is extremely high for practical implementation. Moreover, since inductor L_X carries a dc current, it requires a low-permeability core material or a gap to prevent the core from saturation. As a result, the large number of turns of L_X winding becomes impractical.

Another way to stabilize the power supply is to add a block with a gain K < 1 in the control loop after the EA, as shown in Fig. 10. Then, the original loop gain $T = G_{EA}\cdot G_{PL}$ can be measured as the ratio $\hat{V}_{TEST}/\hat{V}_{REF}$. However, in most power supplies, the EA is located inside the IC controller and the feedback path after the EA cannot be broken physically. Hence, the most practical way to stabilize the power supply is to scale down the EA gain by redesigning the compensation network. The simplest approach to modify the EA transfer function is to add large capacitor C_4 between the EA output and its inverting input, as shown in Fig. 11(a). Figure 11(b) demonstrates the effect of the EA modification on the magnitude plots of transfer function G_{EA} and loop gain T. The frequency of the introduced dominant pole should be low enough to have the loop crossover frequency well below resonant frequency f_0 of the output LC filter. The additional pole changes the EA transfer function, as shown in Fig. 11(b). Observation of Fig. 11(b) reveals that the original loop gain is unstable since its crosses the 0-dB axis at crossover frequency f_{C1} with the –3 slope. The modified loop gain

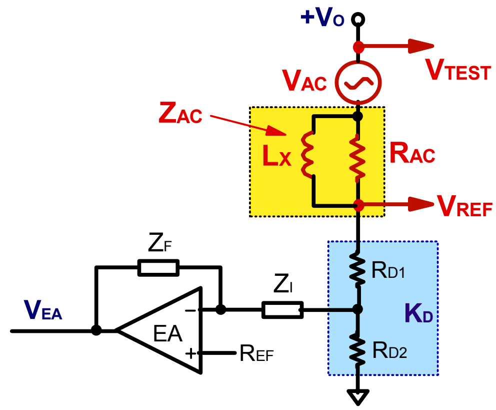


Fig. 9. Loop gain measurement setup for unstable system proposed in [3].

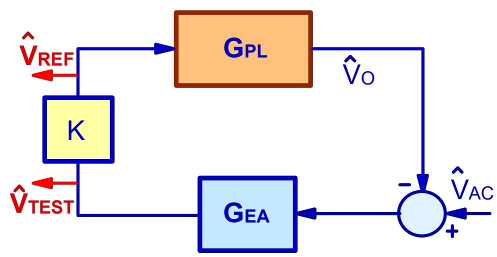
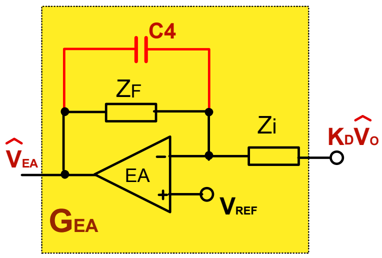


Fig. 10. Stabilization of oscillating power supply by inserting block with gain K < 1.

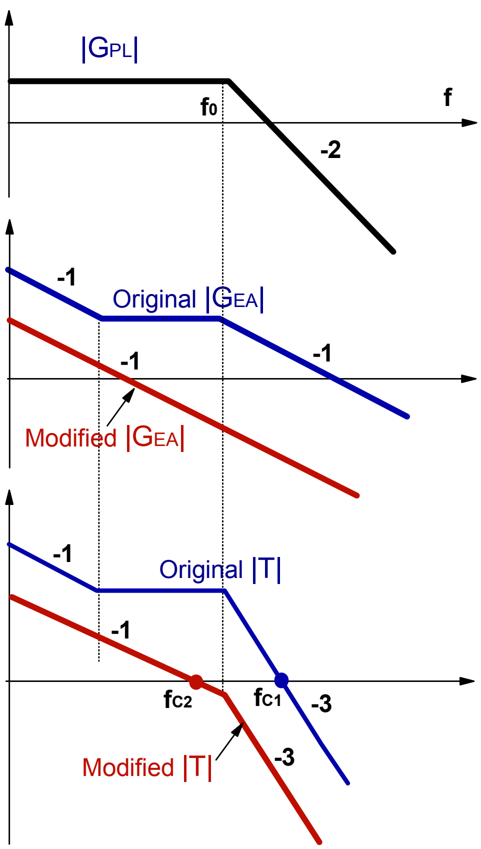
intersects the 0-dB axis at lower frequency f_{C2} with the –1 slope and, therefore, is stable. Unfortunately, only the plant transfer function can be measured by applying this method. To measure the loop gain, the EA Bode plots should be also obtained. However, this task can be accomplished by simulation with the acceptable accuracy. The fact is that from the modeling point of view the EA is a much simpler linear device than the power stage of the switching power supply. Hence, the EA model is often far more accurate than that of the power stage. Still the need to simulate the EA transfer function is the most serious drawback of the proposed approach. For the majority of power supplies, the matching of simulated and measured EA transfer functions is not perfect, but acceptable. The mismatch between the measured and simulated EA Bode plots can be reduced if the EA open-loop gain ***B(s)*** is taken into account

$$G_{EA} = \frac{Z_F}{Z_I + (Z_I + Z_F)/B}. \quad (5)$$

It is usually enough to approximate the open-loop gain with a single-pole transfer function which matches the dc gain and the gain-bandwidth product specified in the manufacturer data sheet.



(a)



(b)

Fig. 11. Stabilization of oscillating power supply by reducing EA gain.

VII. Summary

To satisfy demanding dynamic specifications, the feedback design becomes critical and should be optimized. Optimization requires measurement of the plant transfer function and of the loop gain. The limitations of the conventional closed-loop measurement setup are shown and the measurement procedure, which is free of those limitations, is proposed. The plant identification and EA design approach in several typical power supplies are non-trivial and are discussed in detail. The proposed two-step measurement procedure was successfully applied to the loop gain measurement of the desktop computer power supply with multiple feedback loops. Finally, the loop gain measurement of unstable power supplies was considered.

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